I. (20 *Points*) Find the standard matrix for the stated composition of linear operators:

a. On R^2, a reflection about the x-axis followed by an orthogonal projection on the y-axis. What is the image of the vector (-2; 2) by this composition of linear operators?

b. On R^3, a rotation of 60° about the x-axis, followed by a projection on the yz-plane, followed by a contraction with factor k = ½. . What is the image of the vector (1;1;3)?

II. (20*Points*) Write the polynomial $p = t^2 + 4t - 3$ as a linear combination of the polynomials $p_1 = t^2 - 2t + 5$, $p_2 = 2t^2 - 3t$ and $p_3 = t + 3$

III. (20 *Points*) Determine if the following vectors are linearly dependent. If they are, find a relation between them:

$$\overrightarrow{u_1} = (2;3;7;2),\ \overrightarrow{u_2} = (1;1;2;0),\ \overrightarrow{u_3} = (0;1;1;0) \text{ and } \overrightarrow{u_4} = (2;1;4;1)$$

IV. (20 *Points*) In each part, Determine if V is a vector space or not. In each case precise what are the axioms that fail?

a) $V = \{(x;y) \in R^2 \,/\, 2x + 3y = 0\}$ under the standard addition and scalar multiplication.

b) $$\begin{cases} V = Set\ of\ all\ triples\ of\ real\ numbers(a;b;c), with: \\ \begin{cases} (a;b;c) + (a';b';c') = (a + a';b + b';c + c') & Addition \\ k(a;b;c) = (k^2a;k^2b;k^2c) & Scalar\ Multiplication \end{cases} \end{cases}$$

V. (10 *Points*) Find a unit vector $\vec{u}$ that is orthogonal to both $\vec{v} = 3\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{w} = 4\vec{i} - 2\vec{j} - \vec{k}$

VI. (10 *Points*) Prove the following identities : $(\vec{u} + k\vec{v}) \times \vec{v} = \vec{u} \times \vec{v}$

VII. (**BONUS:** 10 *Points*) Let $f = \cos^2 x$ and $g = \sin^2 x$. Show that the following functions lie in the space spanned by *f* and *g*:

$$h = \cos 2x \qquad h = \frac{3}{2}\cos 2x - \frac{1}{2} \qquad h = 2$$